Quantum and Nuclear Physics (HL)

12.1 The interaction of matter with radiation

Equations

- Planck relationship: $E = hf$ (Energy of a photon at variable frequency) - Relationship between λ & E: $\lambda = \frac{h}{\sqrt{2meV}} \rightarrow \lambda = \frac{h}{E^{\frac{1}{2}}} \rightarrow E = \frac{h}{\lambda^{\frac{1}{2}}} \rightarrow E \propto \lambda^{-\frac{1}{2}} \rightarrow \lambda \propto E^{-\frac{1}{2}}$
- Einstein photoelectric equation: $E_{max} = hf - \Phi$ (the energy required to emit electrons from the metal).
- Bohr orbit energies: $E = -\frac{13.6}{n^2} eV$
- Total energy of electron $= \frac{e^2}{8\pi\varepsilon_0 r} - \frac{e^2}{4\pi\varepsilon_0 r} \rightarrow \frac{-e^2}{8\pi\varepsilon_0 r}$
- Quantization of angular movement: $mvr = \frac{nh}{2\pi}$
- The de Broglie relationship: $\lambda = \frac{h}{\sqrt{2meV}} \rightarrow \lambda = \frac{h}{p} \rightarrow p = \frac{hc}{f}$
- Probability density: $P(r) = |\psi|^2 \Delta V$
- Difference in energy levels: $\Delta E = hf$
- Heisenberg relationships: position-momentum $\rightarrow \Delta x \Delta p \geq \frac{h}{4\pi}$; Energy-Time $\rightarrow \Delta E \Delta t \geq \frac{h}{4\pi}$
- Total energy of an object: $E = \sqrt{p^2c^2 + m_0^2c^4}$

The photoelectric effect

Demonstration of the photoelectric effect

- Can be demonstrated with gold-leaf electroscope.
- The gold leaf photoelectric effect shows how electrons can be ejected when a photon (packet of energy) is emitted from a radioactive source only when it has a certain frequency.
  - Even if light incident on the metal surface is just below required level, it won't eject the electrons. Regardless of intensity.
- The reason that light intensity doesn't affect if the electrons are released or not, even though light intensity = power making it ∝ to energy, is because of photons.
  - Photons energy ∝ its frequency, $E = hf$ (h = planck's constant $6.63 \cdot 10^{-34}$ Js).
- 1 photon interacts with 1 electron.
  - $E \geq$ required to eject 1 electron. (threshold frequency)

If a solar cell is placed in a circuit with a cell, an increase in the pd (0→1V) will have no effect on current, but a negative voltage (0→-1.0V) will affect it as the electrons will have to overcome the pd of the other side of the cell. The point where the electrons with the most E_k are stopped is called the stopping potential. $W = DQ$

- The experiment consists of a charged metal plate mounted on the electroscope plate and the sheet is negatively charged (≈ -3kV).
  - As the photons from radiation are incident on the metal, the electrons are expelled making the leaf collapse to the bottom, as there is less electrostatic repulsion, due to discharge

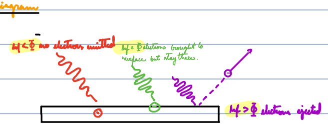
Explanation of photoelectric effect

- The photoelectric effect can be explained in the following ways:
  - Light can be considered to consist of photons, each with energy $= hf$.
  - Each photon can only interact with 1 electron.
  - The minimum photon frequency (threshold frequency (f_0)) is the frequency where any photon with a lower frequency than "f_0" won't emit an electron.
  - The work function (Φ) is the energy needed to do the work to overcome the attractive forces that act on the electrons (electromagnetic) within the metal.
  - If the photon has an energy greater than hf_0, the remainding energy will become the kinetic energy of the emitted electron (often photoelectron).
  - Increasing the intensity of light will only increase the amount of photons incident per second.

Explaining observation from the gold leaf experiment

- The zinc sheet has a certain work function and photons must have a greater energy than this to be able to emit electrons.
- The ultraviolet radiation is used as it has the highest frequency of the radiation.
- With very intense visible or infra-red light incident on the sheet the leaf remains diverged.
  - Increasing the intensity will only increase the number of photons incident per second.
    - The reason they won't eject the electron is because they don't have the required frequency (energy ($E = hf$)).
  - Furthermore, at low intensities, the leaf will collapse instantly as well as there isn't a difference in energy, but rather less photons incident on the metal per second. As soon as a photon with an energy $\geq hf_0$ will instantly eject an electron.
- Placing glass infront of the plate will make the leaf stay up indefinitely as the glass will absorb high energy UV photons, letting low energy ones through.
- If the zinc sheet is charged positively the leaf remains diverged for all wavelengths of radiation
  - Increasing the potential of the metal will increase the energy required to expell one electron.

Diagram



Einstein's photoelectric equation

- $E_{max} = hf - \Phi$
- E_{max} = max kinetic energy of the emitted electron, hf is the energy of the incident photon, and Φ is the work function of the metal.
- It's a max because the work function is defined as the minimum energy required to liberate an electron.
  - Due to the fact that the energy required to remove an electron is given by the formula $E_{max} = hf - \Phi$ when the frequency has reached the threshold frequency (f_0), the value of E will be zero due to the fact that the work function of the metal (Φ) will cancel out the hf_0 making energy 0.

Worked example

- $E_{max} = hf - \Phi$ since at f_0 $E_{max} = 0$

$0 = hf_0 - \Phi$

$\Phi = hf_0$

$\frac{6.6 \cdot 10^{-19}}{6.63 \cdot 10^{-34}} = f_0$

$\underline{f_0 = 1 \cdot 10^{15}\ Hz}$

- $E = hf - \Phi$

$= (6.63 \cdot 10^{-34})(1.2 \cdot 10^{15}) - (6.6 \cdot 10^{-19})$

$= 1.356 \cdot 10^{-19}\ J$

$\underline{\approx 1.4 \cdot 10^{-19}\ J}$

- $2.2\ eV \cdot 1.6 \cdot 10^{-19} = J$

$J = 3.52 \cdot 10^{-19}$

$E = hf$

$2.2\ eV = (6.63 \cdot 10^{-34}) f$

$f = \frac{2.2\ eV}{6.63 \cdot 10^{-34}}$

$f = 5.31 \cdot 10^{14}\ Hz$

- $E = hf - \Phi$

$= (6.63 \cdot 10^{-34})(5.31 \cdot 10^{14}) - (1.5 \cdot 1.6 \cdot 10^{-19})$

$\underline{E = 1.12 \cdot 10^{-19}\ \text{Joules}}$

$k_E = \frac{1}{2} m v^2$

$\sqrt{\frac{2k_E}{m_e}} = v$

$v = \sqrt{\frac{2(1.12 \cdot 10^{-19})}{9.11 \cdot 10^{-31}}}$

$v = 495966.559$

$v \approx 5.00 \cdot 10^5\ ms^{-1}$

- **The wave theory and the photoelectric effect**
  - The reason that wave theory can't explain what is going on is because of the fact that according to wave theory, energy is proportional to the amplitude^2.
    - According to the wave theory, even if a wave of low intensity and any frequency will be able to remove an electron off the surface of the metal overtime. This obviously isn't the case, instead if the frequency of the wave is $f_0 \geq f$, then the electrons won't be expelled.
  - The wave-particle duality simply is refering to light particles acting both as a wave (in diffraction & interference) and as a particle in the photoelectric effect.
- Worked example
  - $E = hf$

    $= (6.6 \cdot 10^{-34})(1.2 \cdot 10^{15})$

    $E = 7.9 \cdot 10^{-19}\ J$
  - $E_{max} = hf - \Phi$

    $= \left(\frac{7.9 \cdot 10^{-19}}{1.6 \cdot 10^{-19}}\right) - 1.8$

    $= \underline{3.2\ eV}$
  - The stopping energy will be the max kinetic energy, 3.2 eV.
- **Matter waves**
  - Matter can also have a wave like properties. The wavelength λ associated with a particle is given by: $\lambda = \frac{h}{p}$
    - λ = wavelength, p = momentum ($p = mv$), and h is Planck's constant ($1.63 \cdot 10^{-34}$).
    - The wavelength is known as the "de Broglie wavelength".
  - The total energy of an object (from the special theory of relativity) is the total of the rest energy and kinetic energy.
    - $E = \sqrt{\underbrace{p^2c^2}_{E_k \text{ term}} + \underbrace{m_0^2c^4}_{\text{Rest mass term}}}$
    - Rest mass is the mass of an object when at rest.
  - Since photons have no rest mass, their energy is: $E = \sqrt{p^2c^2} = pc$, $E = mvc$. For photoelectric effect, $E = \frac{hc}{\lambda}$.
- **Electron diffraction**
  - Electrons will diffract when they pass through a thin film of carbon (crystal). This is a similar property to that of a wave.
  - When electrons are accelerated through a potential difference they gain kinetic energy: $eV \left(= \frac{1}{2} mv^2\right)$
  - Assuming accelerated electrons don't travel close to the speed of light: $p = mv$.
  - The de Broglie relationship: $\lambda = \frac{h}{\sqrt{2 m e V}} \longrightarrow \lambda = \frac{h}{mv} \longrightarrow p = \frac{hf}{c}$
  - Worked example
    - $\lambda = \frac{h}{\sqrt{2 m e V}}$

      $= \frac{1.63 \cdot 10^{-34}}{\sqrt{2(9.1 \cdot 10^{-31})(3000 \cdot 1.6 \cdot 10^{-19})}}$

      $\lambda = 2.24 \cdot 10^{-11}\ m$
  - As the wavelengths of the light passing through a diffraction grating decrease, the diffraction angle (θ) in $n\lambda = d \sin\theta$ will decrease.
- **The Bohr model**
  - Neils Bohr proposed a model of an atom where electrons can only occupy orbitals of a certain radius, based on these assumptions:
    - Electrons in an atom exist in stationary states.
      - Electrons remain in these orbits without emitting any electromagnetic radiation.
    - Electrons can move from orbital to another if they absorb or emit a quantum of electromagnetic radiation (energy).
      - To move from one orbital to another, the electrons have to absorb a quantum amount of energy, and to go down a level the electron will have to release a quantum of radiation.
      - The difference between energy levels is $\Delta E = hf$
    - The angular momentum of an electron in a stationary state is quantized into integral values of $\frac{h}{2\pi}$
      - $mvr = \frac{nh}{2\pi}$
        - n = orbit number, mv = momentum, r = radius
      - Angular momentum is the (vector) product of the momentum of a particle and the radius of its orbit.
- **Energies in the Bohr orbits**
  - For an electron in the nth level (ground state also known as principle quantum number), the total energy E in electronvolts at each level is given by:
    - $E = -13.6$

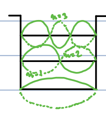
n^2

- The energy is negative because of the fact that energy has to be given to the system in order to completely separate the electron from the proton.

- Worked example
  - The Bohr postulate shows that the electrons can only exist in certain orbitals with a certain amount of energy.
  - The n is the orbital number, the reason that it shows that orbitals are linked to the hydrogen emission spectrum because electrons only exist in orbitals with a finite (quantum) energy, and when electrons move between different energy level the ΔE between the energies (photons) will be emitted.
  - $E = \frac{hc}{\lambda} = \frac{k}{n^2}$ $\quad k = \frac{hc}{0.139\lambda}$

    $= \frac{6.63 \cdot 10^{-34} \cdot 3 \cdot 10^{8}}{362 \cdot 10^{-9} \cdot 0.139}$

    $= \underline{3.95 \cdot 10^{-18} J}$

- Schrödinger's equation
  - Wave-particle duality explains a bright interference fringe as being the place where the probability of finding a particle are high.
    - Described as probability wave
    - Probability curve superposes with one another.
  - Photons incident on the single slit will form a distribution identical to diffraction pattern.
  - Schrödinger's wave function ψ describes the quantum state of particles.] - Explanation
  - For light $I \propto \psi^2$.
  - The $\boxed{\psi}$ may be thought of as the amplitude of the de Broglie wave corresponding to a particle.
    - The de Broglie wave determines the probability of finding the object at a given point.
  - $|\psi|^2$ is proportional to the probability per unit volume of finding the particle, known as probability density: $\boxed{P(r) = |\psi|^2 \Delta V}$.
      - V = volume, r = distance from chosen origin, $P(r)$ is the probability of finding a particle a distance r from a chosen origin.
  - For double slit interference, in terms of the probability wave, the wave function is considered to be such that a single electron will pass through both slits and be everywhere until its observed.
      - When this occurs, the wave function collapses to classical case & particle is detected.
  - The Copenhagen interpretation can be summarised as "nothing is real until its observed".
  - 

    Electrons standing waves in a potential well

      - An electron would be detected somewhere between the nucleus and the outside edge of the atom, shown by the edges of a potential well.
          - Potential varies as the inverse of the distance from the nucleus ($V \propto -\frac{1}{r}$).
  - Within the well, the electron energy must be such that the wave function has nodes at the sides.
  - The probability of finding an electron within the nucleus or outside the atom are zero, so wave amplitude is zero.
      - The electron is most likely found where amplitude is max, halfway between nodes (antinodes).
  - Question:
      - $P(r) = |\psi|^2 \Delta V$ $\quad$ As seen in the graph, the amplitude is at distance a from the origin. The electron will never be found in the nucleus due
        $P(a) = |\psi|^2_{max} \Delta V$ $\quad$ to the fact that $|\psi|^2 = 0$ in nucleus V, the highest probability of finding an electron occurs when the highest value of $|\psi|^2$ occurs due to the fact that $|\psi|^2$ is proportional to the probability per unit volume.

- The Heisenberg uncertainty principle
  - When a quantum is diffracted its only possible to predict its subsequent path in terms of the probability of the wave function.
  - The Heisenberg's uncertainty principle can be written as: $\Delta x \Delta p \geq \frac{h}{4\pi}$.
    - This places a limit on how precisely we are able to know the position and momentum of something in quantum realm.
    - Δx is the uncertainty in the position, while Δp is the uncertainty in the momentum.
  - To find the position of an electron, the principle says there is an uncertainty (given by Δx) with which we can know the position.
    - If Δx is very small, then the uncertainty (Δp) in knowing the momentum is very high.
    - Its impossible to precisely know the position and uncertainty of a electron at the same time. The product of the two uncertainties will always be greater than or equal to $\frac{h}{4\pi}$.
  - If we imagine an electron to be a sine wave (must not be in any field which changes its motion) we can measure its wavelength perfectly.
    - Knowing λ we can determine its momentum ($p = \frac{h}{\lambda}$).
    - Implying that the electron has an infinite uncertainty and is spread out over all of space.
  - To detect a quantum particle its necessary to use something of comparable size to that particle.

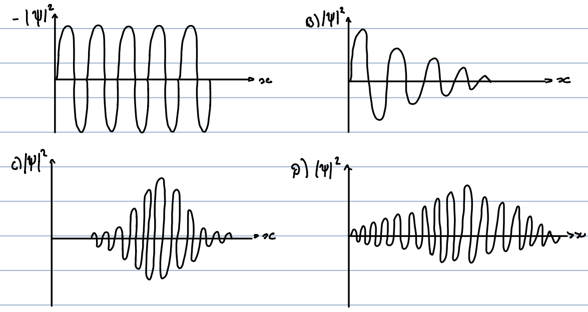
[illegible]

- To do so we would use radiation with $\lambda \approx 10^{-15}$ m.

- From the Broglie relationship we can see that $\lambda = \frac{h}{p}$, and that the larger the momentum, the shorter the λ.
  - meaning $p = \frac{h}{10^{-15}}$
    $= 10^{-18}$ Ns

- When electrons diffract through a narrow gap, maybe $\approx 10^{-18}$ m, the uncertainty principle also applies.
  - when the electron passes through the gap, its uncertainty will be $\pm$ half the gap width.
  - Thus Δp (uncertainty in momentum) $\approx \frac{h}{4\pi \cdot 0.5 \cdot 10^{-15}} \approx \pm 1 \cdot 10^{-16}$ Ns & $\Delta \lambda = \frac{h}{\Delta p} \approx \pm 10^{-17}$ m (ten times gap size).

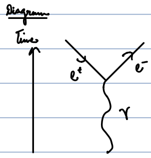
- Worked example



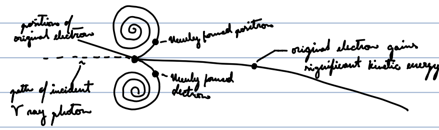
  - The first graph has the highest uncertainty in Δx because of the fact that its waves are the largest meaning that there is an equal probability of finding an electron at each point at the graph. While the other graphs all have a maximum where the probability of finding the electron is higher, meaning $> \Delta x$. As there are more gradual changes in graph D for the momentum, and more lines for the position of the electron, c will have the highest Δp value, as it has the lowest Δx value.

- Pair production & annihilation

  - Pair production is when a photon with sufficient energy (enough energy to form the masses of a particle and its antiparticle) will form a particle & its antiparticle, usually in the vicinity of an atomic nucleus.
    - As a gamma photon passes by a nucleus, the energy of the photon will be converted into electron-positron pair: $\gamma \rightarrow e^- + e^+$.
  - When a photon is close to an atomic nucleus (where the electric field is very strong) with the right energy can turn into a particle along with its antiparticle.
    - This could be an electron and a positron or a proton and an antiproton.
  - The outcome will **always** be a particle and an antiparticle.
    - This is to conserve charge, lepton number, baryon number, and strangeness.
  - The particle and antiparticle are a pair, and the effect is known as pair production.
  - Since the antiparticle is identical to its particle in every way, with the exception of charge, the mass of the two new particles will be equal to one another. Meaning photons must have enough energy to form the two mass, given by:
    - $E = 2mc^2$
      - m is the (rest) mass of the particle/antiparticle (e.g proton ≈ 1.008 u), while c is the speed of an electromagnetic wave in a vacuum ($3 \cdot 10^8$ ms^{-1}).
    - Gamma ray photons can be a method of forming a particle and an antiparticle.
      - Gamma radiation must have at least 1.02 MeV (twice the rest mass of electrons). Any gamma ray photons with $E > 1.02$ MeV will have the excess of energy converted to E_k of the electron-positron pair and the original electron. p. 489
  - Pair production also occurs in the vicinity of an orbital electron, although, more energy will be required to do so as the orbital electron itself gains considerable momentum & kinetic energy. Why does pair production occur and why near the nucleus?
  - Diagram



Electron - positron pair production
  - The diagram shows pair production taking place near an electron.
  - The photon is non-ionising, but an electron and positron are seen spiralling in opposite directions in the applied magnetic field.
- The recoiling electron gains a lot of E_k, meaning it won't bend a lot in the magnetic field.
  - Although, it will bend in the magnetic field.
- Theory shows that the threshold energy needed for this type of pair production is $4mc^2$ (2.04 MeV)

- Equation: $\gamma + e^- \rightarrow e^- + e^- + e^+$
  - And is it necessary for the pair production?
- When a particle and its antiparticle encounter they annihilate, forming two photons.
  - The energy of the photons is equal to the total mass-energy of the annihilating particles.
- Sometimes when a pair annihilates, a photon or both photons that the pair produce when they annihilate can be converted into a electron-positron pair which then go onto annihilating back into two photons.
- The positron quickly is annihilated by another electron in matter.
- Electron-positron pair production
  - 

- Pair production and the Heisenberg uncertainty principle
  - The Heisenberg uncertainty principle states that you'll never be able to know what the exact momentum and position of a particle at an exact instant in time. A particle doesn't have a position (x) & momentum (p) simultaneously.
  - Other factors other than momentum and position are a factor to the uncertainty of the Heisenberg principle.
    - These factors being energy and time which are conjugate variables. The relationship of time & energy to Heisenberg's uncertainty principle are:
      - $\Delta E \Delta t \geq \frac{h}{4\pi}$
        - ΔE represents the uncertainty in the energy, and Δt is the uncertainty in of the lifetime of the pair.
  - The threshold frequency required for the production of an electron-positron pair is lower than 1.02 MeV when the gamma photon is near a heavy nucleus e.g 10 eV.
    - Once the gamma photon has formed an electron & a positron, they're going to annihilate to then form two photons of 5 eV photons.
      - This is allowable under the uncertainty principle.
  - During the lifetime of the composition of an electron-positron pair there is an uncertainty regarding the total energy.
  - Example calculation:
    - $\Delta E = 1.02$ MeV, find Δt
      - $\Delta E \Delta t = \frac{h}{4\pi}$

        $\Delta t = \frac{(6.63 \cdot 10^{-34})}{(4\pi)(\underbrace{1.02 \cdot 10^{6} \cdot 1.6 \cdot 10^{-19}}_{\text{Conversion from MeV to Joules}})}$

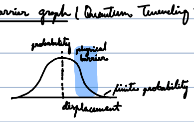
        $\underline{\Delta t = 3.2 \cdot 10^{-22}\,s}$
  - The lifetime is so short that the energy of the pair would have an uncertainty of at least 1.02 MeV and the experiment wouldn't be in violation of the law of conservation of energy.
- Quantum Tunneling
  - A particle's wave function has a finite probability of being everywhere in the universe at the same time.
    - An example of this would be that of a golf ball is in a hole, it can't get out, but using Heisenberg's uncertainty principle ($\Delta E \Delta t \geq \frac{h}{4\pi}$) says that if the energy is borrowed for a short enough time then the ball could "tunnel out" of the hole, returning the energy when it moves down the hill.

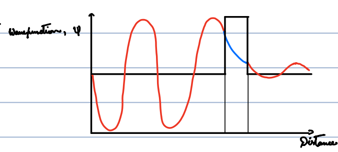
Solar constant $= 1.0 \cdot 10^{3}\ Wm^{-2}$

$f = \frac{c}{500 \cdot 10^{9}}$

$E = \frac{hc}{\lambda}$

$\frac{nE}{t} = IA$

    - This means that an electron in the ground state of hydrogen could escape without having to do 13.6 eV of work.
  - The energy required to overcome the energy barrier of something like a ball getting out of a hole is so large that the time has to be so low that the probability of this occurring is infinitely small.
    - Although, with an electron it's much more possible as the energy required is lower with the lower mass.
  - Potential Barrier graph (Quantum Tunneling through physical barrier)
    - 

    - 

      - As seen the amplitude after the potential barrier isn't zero, showing that while the probability ($P(r) = |\Psi|^2 \Delta V$) of finding an electron there is very low, but not impossible.
        - As stated in the section about the Schrödinger equation, the larger the amplitude, the higher the probability of finding an electron.
  - The likelihood of tunneling occurring is affected by the following factors:

- Thickness of barrier
  - The further the particle has to tunnel, the smaller the amplitude of the wave function on the other side, making tunneling less likely.
- Amount of energy required
  - The larger the energy required, the less likely tunneling is to occur.
    - An example of a barrier is alpha decay, which has to overcome an energy barrier (not a physical one). The greater the amount of energy required by the particle to overcome the barrier, the less likely tunneling is to occur.
- Mass
  - The more massive the particle is, the less likely it is for the particle to tunnel through a barrier. Due to the mass-energy relationship.

- Using the uncertainty principle a particle can use the energy from its surroundings to escape the nucleus, and then pay the energy back as long as it doesn't take too long.
- Quantum tunneling is used a lot in the sun, and is responsible for the relatively low temperatures for fusion in the sun.
  - The repulsive forces between the protons means that the energy required for fusion is $\simeq 1\,MeV = 10^{10}\,K$, yet fusion occurs when the core temp of the sun $\simeq 3 \cdot 10^{7}\,K$.
  - This is due to the fact that some hydrogen nuclei are able to overcome the energy barrier & fuse at temps below $10^{10}\,K$. Meaning that fusion will occur.
- Worked example
  - Ψ is the amplitude of the de Broglie wave corresponding to a particle, while $|\psi|^2$ is the probability of finding an electron in a certain volume, given by the equation: $P(r) = |\psi|^2 \Delta V$.
  - $p = \frac{(6.63 \cdot 10^{-34})}{\lambda}$ $\quad \lambda = \frac{2 \cdot 10^{-10}}{6}$

    $= \frac{(6.63 \cdot 10^{-34})}{3.3 \cdot 10^{-11}}$ $\quad \lambda = 3.3 \cdot 10^{-11}\,m$

    $= 2 \cdot 10^{-23}\,Ns$
  - $\Delta p = \frac{h}{4\pi \Delta x}$

    $= \frac{(6.63 \cdot 10^{-34})}{4\pi(2 \cdot 10^{-10})}$

    $= 2.6 \cdot 10^{-25}\,Ns$